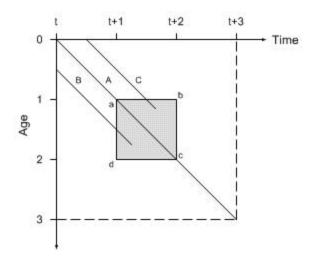
## Encyclopedia of Population Lexis Diagrams

## Griffith Feeney Final Prepublication Version

Lexis diagrams play a valuable role in demographic analysis by providing a highly effective visual language for conveying information about the sets of persons and events that are the basis of all population statistics. The diagrams complement verbal descriptions of these sets, which are often clumsy and hard to grasp. The diagrams are named after the German statistician and actuary Wilhelm Lexis (1837–1914).

There are four principles of Lexis diagram representation. *First*, a demographic event may be represented by a point on a coordinate plane whose coordinates are the time at which the event occurred and the age of the person to whom it occurred. *Second*, each two dimensional set in the coordinate plane represents the set of events whose representing points fall within its boundaries. *Third*, a person may be represented by the straight line, called a *life line*, connecting the points representing this person's birth and death. *Fourth*, each line in the coordinate plane represents the set of persons whose life line intersect this line.

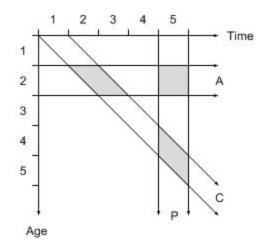
The following figure illustrates each of these principles. Line A is the life line of a person



who was born at time *t* and died, at exact age 3, at time t+3. Lines B and C both represent persons who died during year t+1 at 1 completed year of age (i.e., at an exact age  $\ge 1$  and <2). Square *abcd* represents the set of all such deaths, which divide into two parts: deaths of persons who were age 1 in completed years at the beginning of year t+1, represented by triangle *acd*, and deaths of persons who reached exact age 1 during that year, represented by triangle *abc*. The set of persons who reached exact age 1 (celebrated their first birthday) during year t+1 is represented by line *ab*. The set of persons who were age 1 in completed years at the beginning of year t+1 is represented by line *ad*.

The drawing and interpretation of Lexis diagrams is facilitated by two general methods. The *method of extremes* consists of identifying extreme cases, drawing lines or plotting points representing these cases, and connecting these points or lines to obtain the desired representation. Referring to the above figure, for example, consider the set of persons who reach exact age 1 during year t+1. The extreme cases here are reaching exact age 1 at time t+1 and reaching exact age 1 at time t+2, corresponding to points a and b, respectively. The line ab connecting these points is the desired representation.

The *method of intersections*, which applies only to sets of events, consists of identifying an age group, time period, and/or birth cohort, drawing the representations of the groups so identified, and taking the intersection of these representations. Three examples of practical importance are illustrated in the following figure, in which age groups and time periods of equal length have been marked off.



Deaths occurring during the 5th time period to persons in the 2nd age group are represented by the square, which is the intersection of the horizontal strip A corresponding to the age group and the vertical strip P corresponding to the time period. Numbers of deaths in such sets are the numerators of age-specific death rates.

Deaths of persons in the cohort born in the 1st period that occur when these persons are in the 2nd age group are represented by the parallelogram with sides parallel to the time axis, which is the intersection of the horizontal strip A representing the age group and the diagonal strip C representing the cohort. Numbers of deaths in such sets are the numerators of life table "probabilities" of death.

Deaths of persons in the cohort born during the 1st period that occur during the 5th period are represented by the parallelogram with sides parallel to the age axis, which is the intersection of the diagonal cohort strip C and the vertical time period strip P. Numbers of deaths in such sets figure in population projection calculations.

The age axis in the Lexis diagram may be replaced by an axis representing time elapsed since any event, such as marriage, divorce or first birth, providing for description of a wider variety of sets of persons and events. The time axis most often represents calendar time, but diagrams for sample survey data may express time as months or years prior to interview. Because the metric of age is elapsed time, the units of time and age are represented by the same distance on the two axes. It follows that the life lines form a 45degree angle to the axes.

Lexis diagram representations apply to events of all kinds, including births regarded as events occurring to the mother, marriages, and divorces. The diagrams themselves do not indicate what kind of event is represented, however. This information must be supplied by context. The point representing any event occurring to a person necessarily lies on the life line representing this person. This obvious but important fact serves to identify persons experiencing particular events as members of various groups, for example, as members of a particular birth cohort.

Life lines may be generalized to represent a person's membership in a particular population by removing from the line those points corresponding to periods when the person was not a member of the population. These generalized life lines may be used, for example, to represent persons in populations that experience migration.

Different orientations of the Lexis diagram axes may be used for different purposes. The orientation shown above is most generally useful because it corresponds to the way tables of births and deaths are arranged, with rows for events occurring at older ages placed below rows for events occurring at younger ages.

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